Kanyakumari District Common Examination
Common Model Examination, January - 2015

Standard 12
MATHEMATICS

Time: 3.00 Hrs. Marks: 200

Section - A
40×1=40

Note: (i) All questions are compulsory. (ii) Choose the correct answer.

1) The rank of the $\begin{bmatrix} -1 & & & \\ & 2 & & \\ & & 0 & \\ & & & -4 \\ & & & & 0 \end{bmatrix}$ diagonal matrix?

a) 0 b) 2 c) 3 d) 5

2) If the rank of the matrix $\begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -1 & 0 & \lambda \end{bmatrix}$ is 2, then λ is

a) 1 b) 2 c) 3 d) any real number

3) If I is the unit matrix of order n, where $K \neq 0$ is a constant, then adj (KI) =

a) K^n(adj I) b) K(adj I) c) K^2(adj (I)) d) K^{n-1} (adj I)

4) Which of the following is not elementary transformation?

a) $R_i \leftrightarrow R_j$ b) $R_i \rightarrow 2R_i+R_j$ c) $C_i \rightarrow C_j+C_i$ d) $R_i \rightarrow R_i+C_j$

5) If $\vec{a}$ and $\vec{b}$ are two unit vectors and θ is the angle between them, then $\left(\vec{a}+\vec{b}\right)$ is a unit vector if

a) $\theta = \pi/3$ b) $\theta = \pi/4$ c) $\theta = \pi/2$ d) $\theta = 2\pi/3$

6) If $\vec{a}+\vec{b}+\vec{c} = 0$, $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$ then the angle between $\vec{a}$ and $\vec{b}$ is

a) $\pi/6$ b) $2\pi/3$ c) $5\pi/3$ d) $\pi/2$

7) If $\vec{a}\times\left(\vec{b}\times\vec{c}\right)+\vec{b}\times\left(\vec{c}\times\vec{a}\right)+\vec{c}\times\left(\vec{a}\times\vec{b}\right) = \vec{x}\times\vec{y}$ then

a) $\vec{x} = \vec{0}$ b) $\vec{y} = \vec{0}$ c) $\vec{x}$ and $\vec{y}$ are parallel

d) $\vec{x} = \vec{0}$ or $\vec{y} = \vec{0}$ or $\vec{x}$ and $\vec{y}$ are parallel

8) If $\vec{a}\times\left(\vec{b}\times\vec{c}\right) = \left(\vec{a}\times\vec{b}\right)\times\vec{c}$ for non coplanar vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ then

a) $\vec{a}$ parallel to $\vec{b}$ b) $\vec{b}$ parallel to $\vec{c}$ c) $\vec{c}$ parallel to $\vec{a}$ d) $\vec{a}+\vec{b}+\vec{c} = \vec{0}$

9) The vector equation of a plane passing through the line of intersection of the planes $\vec{r}.\vec{n}_1 = q_1$ and $\vec{r}.\vec{n}_2 = q_2$ is

a) $\left(\vec{r}.\vec{n}_1 - q_1\right)+\lambda\left(\vec{r}.\vec{n}_2 - q_2\right) = 0$ b) $\vec{r}.\vec{n}_1+\vec{r}.\vec{n}_2 = q_1+\lambda q_2$

c) $\vec{r}\times\vec{n}_1+\vec{r}\times\vec{n}_2 = q_1+q_2$ d) $\vec{r}\times\vec{n}_1-\vec{r}\times\vec{n}_2 = q_1+q_2$

10) If $x^2+y^2 = 1$ then the value of $\dfrac{1+x+iy}{1+x-iy}$ is

a) x−iy b) 2x c) −2iy d) x+iy

11) If z represents a complex number then arg (z) + arg $\left(\bar{Z}\right)$ is

a) $\pi/4$ b) $\pi/2$ c) 0 d) $3\pi/4$

12) $\dfrac{1+e^{-i\theta}}{1+e^{i\theta}}$ =

a) cos θ + i sin θ b) cos θ − i sin θ c) sin θ − i cos θ d) sin θ + i cos θ

13) The value of $\bar{\bar{Z}}+\bar{Z}$ is

a) 2 Re (Z) b) Re (Z) c) Im (Z) d) 2Im (Z)

14) The line 4x+2y = c is a tangent to the parabola $y^2 = 16x$ then c is

a) −1 b) −2 c) 4 d) −4

15) The focus of the parabola $x^2 = 16y$ is
a) (4, 0) b) (0, 4) c) (−4, 0) d) (0, −4)

16) The sum of the distance of any point on the ellipse $4x^2+9y^2 = 36$ from $(\sqrt{5}, 0)$ and $(-\sqrt{5}, 0)$ is
a) 4 b) 8 c) 6 d) 18

17) The angle between the asymptotes to the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ is
a) $\pi - 2\tan^{-1}(3/4)$ b) $\pi - 2\tan^{-1}(4/3)$ c) $2\tan^{-1}(3/4)$ d) $2\tan^{-1}(4/3)$

18) The normal to the rectangular hyperbola $xy = 9$ at $(6, 3/2)$ meets the curve again at
a) $(3/8, 24)$ b) $(-24, -3/8)$ c) $(-3/8, -24)$ d) $(24, 3/8)$

19) The gradient of the curve $y = -2x^3+3x+5$ at $x = 2$ is
a) −20 b) 27 c) −16 d) −21

20) The slope of the tangent to the curve $y = 3x^2+3 \sin x$ at $x = 0$
a) 3 b) 2 c) 1 d) −1

21) The curve $y = ax^3+bx^2+cx+d$ has a point of inflexion at $x = 1$ then
a) a+b = 0 b) a+3b = 0 c) 3a+b = 0 d) 3a+b = 1

22) If a real valued differentiable function $y = f(x)$ defined on an open interval I is increasing then
a) $dy/dx > 0$ b) $dy/dx \geq 0$ c) $dy/dx < 0$ d) $dy/dx \leq 0$

23) If $u = \log\left(\frac{x^2+y^2}{xy}\right)$ then $x\frac{du}{dx} + y\frac{du}{dy}$ is
a) 0 b) u c) 2u d) u^{-1}

24) The curve $y^2 = (x-1)(x-2)^2$ is not defined for
a) $x \geq 1$ b) $x \geq 2$ c) $x < 2$ d) $x < 1$

25) The value of $\int_0^{\pi} \sin^2 x \cos^3 x \, dx$ is
a) π b) π/2 c) π/4 d) 0

26) The area bounded by the parabola $y^2 = x$ and its latus rectum is
a) 4/3 b) 1/6 c) 2/3 d) 8/3

27) The length of the arc of the curve $x^{2/3} + y^{2/3} = 4$ is
a) 48 b) 24 c) 12 d) 96

28) $\int_0^{\infty} x^6 e^{-x/2} \, dx =$
a) $\lfloor 6 / 2^7$ b) $\lfloor 6 / 2^6$ c) $2^6 \lfloor 6$ d) $2^7 \lfloor 6$

29) The integrating factor of the differential equation $\frac{dy}{dx} + Py = Q$
a) $\int P \, dx$ b) $\int Q \, dx$ c) $e^{\int Q \, dx}$ d) $e^{\int P \, dx}$

30) The differential equation satisfied by all the straight lines in xy plane is
a) $\frac{dy}{dx}$ = a constant b) $\frac{d^2y}{dx^2} = 0$ c) $y + \frac{dy}{dx} = 0$ d) $\frac{d^2y}{dx^2} + y = 0$

31) The particular integral of the differential equation $f(D)y = e^{ax}$ where $f(D) = (D-a)\, g(D)$, $g(a) \neq 0$ is
a) me^{ax} b) $\frac{e^{ax}}{g(a)}$ c) $g(a)\, e^{ax}$ d) $\frac{xe^{ax}}{g(a)}$

32) The order and degree of the differential equation $\frac{d^2y}{dx^2} + x = \sqrt{y + \frac{dy}{dx}}$ are
a) 2, 1 b) 1, 2 c) 2, ½ d) 2, 2

33) $p \leftrightarrow q$ is equivalent to
a) $p \leftrightarrow q$ b) $q \leftrightarrow p$ c) $(p \rightarrow q) \vee (q \rightarrow p)$ d) $(p \rightarrow q) \wedge (q \rightarrow p)$

34) The order of (7) in $(Z_9, +_9)$ is
a) 9 b) 6 c) 3 d) 1

35) In the set of integers under the operation * defined by a*b = a+b−1, the identity element is
a) 0 b) 1 c) a d) b

36) In the group (G_2), G = (1, −1, i, −i) the order of 1 is
a) 2 b) 0 c) 4 d) 1

37) Variance of random variable X is 4. Its mean is 2. Then $E(x^2)$ is
a) 2 b) 4 c) 6 d) 8

38) Given E (X+c) = 8 and E(X−C) = 12, then the value of C is
a) −2 b) 4 c) −4 d) 2

39) In a poisson distribution if P(X = 2) = P (X = 3) then the value of its parameter λ is
a) 6 b) 2 c) 3 d) 0

40) If X is a discrete random variable then which of the following is correct?
a) $0 \le F(x) < 1$ b) $F(-\infty) = 0$ and $F(\infty) \le 1$
c) $P(X = X_n) = F(x_n) - F(x_n - 1)$ d) F(x) is a constant function

Section - B

10×6=60

Note: (i) Answer any ten questions.
(ii) Question no. 55 is compulsory and choose any nine from the remaining.

41) Find the rank of $\frac{1}{3}\begin{pmatrix} -1 & 3 & 3 & 1 \\ -0 & 3 & 6 & 3 \\ 3 & -3 & 6 & 0 \end{pmatrix}$

42) If $A = \begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix}$, then find adj (A) and also verify A(adj (A)) = (adj (A))A = $|A|I_2$

43) Show that the diameter of a sphere subtends a right angle at a point on the surface.

44) a] A force $4\vec{i} + 2\vec{j} + \vec{k}$ acting on a point B (5, 2, 4) find the moment of the force through the point A(3, −1, 3)

b] Find the value of λ when the planes $\vec{r}.(2\vec{i} + \lambda\vec{j} - 3\vec{k}) = 10$ and $\vec{r}.(\lambda\vec{i} + 3\vec{j} + \vec{k}) = 5$ are perpendicular.

45) Prove that |Z| = 1 when the argument of (Z−1) is $\pi/6$ and (Z+1) is $2\pi/3$

46) If $x + \frac{1}{x} = 2\cos\theta$ then prove that

(i) $x^n + \frac{1}{x^n} = 2\cos n\theta$ and (ii) $x^n - \frac{1}{x^n} = 2i\sin n\theta$

47) Prove that the product of perpendiculars from any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to its asymptotes is constant and the value is $\frac{a^2b^2}{a^2 + b^2}$

48) Show that the curves $2x^2+4y^2 = 1$ and $6x^2-12y^2 = 1$ are perpendicular to each other.

49) Obtain the maclaurin's series for $\tan^{-1}x$.

50) Evaluate: $\int_0^1 \log\left(\frac{1}{x} - 1\right) dx$

51) Solve: $\frac{dy}{dx} + y\cot x = 2\cos x$

52) Use the truth table to determine $(p\wedge(\sim q))\vee((\sim p)\vee q)$ is a tautology or a contradiction.

53) Prove that $(Z_5-\{[0]\}, \cdot_5)$ is a group

54) The probability of success of an event is p and that of failure is q. Find the expected number of trails to get a frist success.

55) a] Use differentials to find an approximate value for $\sqrt{36.1}$ for two decimal places.

(OR)

b] Verify that the following are probability density function.

(i) $f(x) = \begin{cases} 2x/9, & 0 \le x \le 3 \\ 0, & elsewhere \end{cases}$ (ii) $f(x) = \frac{1}{\pi}\frac{1}{(1+x^2)}, \; -\infty < x < \infty$

Section - C **10×10=100**

Note: (i) Answer any ten questions.
(ii) Question no. 70 is compulsory and choose any nine questions from the remaining.

56) For what value of K, the system of equations Kx+y+z = 1; x+Ky+z = 1; x+y+Kz = 1 have (i) unique solution (ii) more than one solution (iii) no solution

57) Altitudes of a triangle are concurrent prove by vector method.

58) Find the vector and cartesian equations of the plane passing through the points (−1, 1, 1) and (1, −1, 1) and perpendicular to the plane x+2y+2z = 5.

59) Solve: $x^4-x^3+x^2-x+1 = 0$

60) An arch is in the form of a semi ellipse whose span is 48 ft wide. The height of the arch is 20 feet. How wide is the arch at a height of 10 feet above the base?

61) Find the eccentricity, centre, focii and vertices of the hyperbola $x^2-4y^2+6x+16y-11 = 0$ and also trace the curve.

62) A water tank has the shape of an inverted circular cone with base radius 2 metres and height 4 metres. If water is being pumped into the tank at a rate of $2m^2$/min. find the rate at which the water level is rising when the water is 3 m deep.

63) Find the points of inflection and determine the intervals of convexity and concavity of the Gaussion curve $y = e^{-x^2}$

64) Trace the curve $y = x^3$.

65) Find the area bounded by x-axis and an arch of the cycloid x = a (2t− sin 2t) and y = a(1−cos 2t)

66) Solve the differential equation $(1+2x^3)\frac{dy}{dx} + 6x^2y = \cos ec^2 x$

67) The rate at which the population of a city increases at any time is proportional to the population at that time. If there were 1,30,000 people in the city in 1960 and 1,60,000 in 1990 what population may be anticipated in 2020.

$\left[\log_e\left(\frac{16}{13}\right) = 0.2070;\; e^{0.42} = 1.52\right]$

68) Show that (Z, *) is an infinite abelian group where * is defined as a*b = a+b+2.

69) Obtain K, μ and σ^2 of the normal distribution whose probability distribution function is given by $f(x) = Ke^{-2x^2+4x}, \; -\infty < x < \infty$

70) a] Find the surface area of the solid generated by revolving the arch of the parabola $y^2 = 4ax$ bounded by its latus rectum.

(OR)

b] Find the equation of the rectangular hyperbola which has for one of its asymptotes the line x+2y−5 = 0 and passes through the point (6, 0) and (−3, 0)

-x-x-x-x-