Part III

MATHEMATICS

(தமிழ் பதிவு எளிதான மாதிரி / Tamil & English Versions)

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Register Number

Time Allowed : 3 Hours |

Maximum Marks : 200

A

Instruction : Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

III - A / SECTION - A

Note : i) All questions are compulsory.
ii) Choose the most suitable answer from the given four alternatives.

40 x 1 = 40

1. \( \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + \mathbf{b} \times (\mathbf{c} \times \mathbf{a}) + \mathbf{c} \times (\mathbf{a} \times \mathbf{b}) = \mathbf{x} \times \mathbf{y} \) என்பிடும்,
   1) \( \mathbf{x} = 0 \)
   2) \( \mathbf{y} = 0 \)
   3) \( \mathbf{x} \parallel \mathbf{y} \) என்பது உறாதாரம்
   4) \( \mathbf{x} = 0 \) உறாவது \( \mathbf{y} = 0 \) உறாவது \( \mathbf{x} \parallel \mathbf{y} \) என்பது உறாதாரம்

If \( \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + \mathbf{b} \times (\mathbf{c} \times \mathbf{a}) + \mathbf{c} \times (\mathbf{a} \times \mathbf{b}) = \mathbf{x} \times \mathbf{y} \), then
   1) \( \mathbf{x} = 0 \)
   2) \( \mathbf{y} = 0 \)
   3) \( \mathbf{x} \) and \( \mathbf{y} \) are parallel
   4) \( \mathbf{x} = 0 \) or \( \mathbf{y} = 0 \) or \( \mathbf{x} \) and \( \mathbf{y} \) are parallel.
2. \[3 \hat{i} + \hat{j} - \hat{k} \text{ and } \hat{i} - 3 \hat{j} + 4 \hat{k}\] are \(\vec{a}\) and \(\vec{b}\) respectively, find \(\vec{a} \cdot \vec{b}\).

   \[\begin{align*} 1) & \ 10\sqrt{3} & 2) & \ 6\sqrt{30} & 3) & \ \frac{3}{2}\sqrt{30} & 4) & \ 3\sqrt{30} \end{align*}\]

The area of the parallelogram having a diagonal \(3 \hat{i} + \hat{j} - \hat{k}\) and a side \(\hat{i} - 3 \hat{j} + 4 \hat{k}\) is

1) \(10\sqrt{3}\) 2) \(6\sqrt{30}\) 3) \(\frac{3}{2}\sqrt{30}\) 4) \(3\sqrt{30}\).

3. \(\vec{r} = (- \hat{i} + 2 \hat{j} + 3 \hat{k}) + t(-2 \hat{i} + \hat{j} + \hat{k})\) and \(\vec{r} = (2 \hat{i} + 3 \hat{j} + 5 \hat{k}) + s(\hat{i} + 2 \hat{j} + 3 \hat{k})\) are two lines. Find the point of intersection of the lines.

1) \((2, 1, 1)\) 2) \((1, 2, 1)\) 3) \((1, 1, 2)\) 4) \((1, 1, 1)\).

The point of intersection of the lines \(\vec{r} = (- \hat{i} + 2 \hat{j} + 3 \hat{k}) + t(-2 \hat{i} + \hat{j} + \hat{k})\) and \(\vec{r} = (2 \hat{i} + 3 \hat{j} + 5 \hat{k}) + s(\hat{i} + 2 \hat{j} + 3 \hat{k})\) is

1) \((2, 1, 1)\) 2) \((1, 2, 1)\) 3) \((1, 1, 2)\) 4) \((1, 1, 1)\).

4. \(2 \hat{i} - \hat{j} + 5 \hat{k}\) and \(\hat{i} + 2 \hat{j} - 2 \hat{k}\) are \(\vec{a}\) and \(\vec{b}\) respectively.

1) \(-\frac{10}{\sqrt{30}}\) 2) \(\frac{10}{\sqrt{30}}\) 3) \(\frac{1}{3}\) 4) \(\frac{\sqrt{10}}{30}\).

The projection of \(\hat{i} + 2 \hat{j} - 2 \hat{k}\) on \(2 \hat{i} - \hat{j} + 5 \hat{k}\) is

1) \(-\frac{10}{\sqrt{30}}\) 2) \(\frac{10}{\sqrt{30}}\) 3) \(\frac{1}{3}\) 4) \(\frac{\sqrt{10}}{30}\).

5. \(\vec{r} \cdot (3 \hat{i} + 4 \hat{j} + 12 \hat{k}) = 26\) is the equation of a plane. Find the length of the perpendicular from the origin to the plane.

1) \(26\) 2) \(\frac{26}{169}\) 3) \(2\) 4) \(\frac{1}{2}\).

The length of the perpendicular from the origin to the plane \(\vec{r} \cdot (3 \hat{i} + 4 \hat{j} + 12 \hat{k}) = 26\) is

1) \(26\) 2) \(\frac{26}{169}\) 3) \(2\) 4) \(\frac{1}{2}\).
6. \( xy = 32 \)  
\[ \begin{array}{llll} 
1) & 8 \sqrt{2} & 2) & 32 \ & 3) & 8 \ & 4) & 16. 
\end{array} \]

The length of the latus rectum of the rectangular hyperbola \( xy = 32 \) is 
\[ \begin{array}{llll} 
1) & 8 \sqrt{2} & 2) & 32 \ & 3) & 8 \ & 4) & 16. 
\end{array} \]

7. \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \)  
\[ \begin{array}{llll} 
1) & \frac{x^2}{144} - \frac{y^2}{432} = 1 
2) & \frac{x^2}{432} - \frac{y^2}{144} = 1 
3) & \frac{x^2}{12} - \frac{y^2}{12 \sqrt{3}} = 1 
4) & \frac{x^2}{12 \sqrt{3}} - \frac{y^2}{12} = 1. 
\end{array} \]

The difference between the focal distances of any point on the hyperbola \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \) is 24 and the eccentricity is 2. Then the equation of the hyperbola is 
\[ \begin{array}{llll} 
1) & \frac{x^2}{144} - \frac{y^2}{432} = 1 
2) & \frac{x^2}{432} - \frac{y^2}{144} = 1 
3) & \frac{x^2}{12} - \frac{y^2}{12 \sqrt{3}} = 1 
4) & \frac{x^2}{12 \sqrt{3}} - \frac{y^2}{12} = 1. 
\end{array} \]

8. \( y = mx + c \)  
\[ \begin{array}{llll} 
1) & \left( \begin{array}{c} b^2 \\ a^2 m \\ c \end{array} \right) 
2) & \left( \begin{array}{c} -a^2 m \\ b^2 \\ c \end{array} \right) 
3) & \left( \begin{array}{c} a^2 m \\ -b^2 \\ c \end{array} \right) 
4) & \left( \begin{array}{c} -a^2 m \\ -b^2 \\ c \end{array} \right). 
\end{array} \]

The point of contact of the tangent \( y = mx + c \) and the ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) is 
\[ \begin{array}{llll} 
1) & \left( \begin{array}{c} b^2 \\ a^2 m \\ c \end{array} \right) 
2) & \left( \begin{array}{c} -a^2 m \\ b^2 \\ c \end{array} \right) 
3) & \left( \begin{array}{c} a^2 m \\ -b^2 \\ c \end{array} \right) 
4) & \left( \begin{array}{c} -a^2 m \\ -b^2 \\ c \end{array} \right). 
\end{array} \]
9. \( f(x) = x^2 - 5x + 4 \) is increasing in
1) \( (-\infty, 1) \)
2) \( (1, 4) \)
3) \( (4, \infty) \)
4) everywhere.

The function \( f(x) = x^2 - 5x + 4 \) is increasing in
1) \( (-\infty, 1) \)
2) \( (1, 4) \)
3) \( (4, \infty) \)
4) everywhere.

10. \( a = 1 \) and \( b = 4 \) for \( f(x) = \sqrt{x} \) is increasing in
1) \( \frac{9}{4} \)
2) \( \frac{3}{2} \)
3) \( \frac{1}{2} \)
4) \( \frac{1}{4} \).

The value of \( c \) of Lagrange's mean value theorem for \( f(x) = \sqrt{x} \) when \( a = 1 \) and \( b = 4 \) is
1) \( \frac{9}{4} \)
2) \( \frac{3}{2} \)
3) \( \frac{1}{2} \)
4) \( \frac{1}{4} \).

11. The volume generated when the region bounded by \( y = x, y = 1 \) is rotated about \( y \)-axis is
1) \( \frac{\pi}{4} \)
2) \( \frac{\pi}{2} \)
3) \( \frac{\pi}{3} \)
4) \( \frac{2\pi}{3} \).

12. \( \int_0^\pi \sin^4 x \, dx \) is
1) \( \frac{3\pi}{16} \)
2) \( \frac{3}{16} \)
3) \( 0 \)
4) \( \frac{3\pi}{8} \).

The value of \( \int_0^\pi \sin^4 x \, dx \) is
1) \( \frac{3\pi}{16} \)
2) \( \frac{3}{16} \)
3) \( 0 \)
4) \( \frac{3\pi}{8} \).
13. \( I_n = \int \sin^n x \, dx \)  

1) \( - \frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} I_{n-2} \)  
2) \( \frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} I_{n-2} \)  
3) \( - \frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} I_{n-2} \)  
4) \( \frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} I_{n-2} \)  

If \( I_n = \int \sin^n x \, dx \), then \( I_n \) is:

1) \( - \frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} I_{n-2} \)  
2) \( \frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} I_{n-2} \)  
3) \( - \frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} I_{n-2} \)  
4) \( \frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} I_{n-2} \)  

14. \( \frac{dy}{dx} + \frac{2y}{x} = e^{4x} \)  

1) \( \log x \)  
2) \( x^2 \)  
3) \( e^x \)  
4) \( x \)  

The integrating factor of \( \frac{dy}{dx} + \frac{2y}{x} = e^{4x} \) is:

1) \( \log x \)  
2) \( x^2 \)  
3) \( e^x \)  
4) \( x \)  

15. \( (3D^2 + D - 14)y = 13e^{2x} \)  

1) \( 26xe^{2x} \)  
2) \( 13xe^{2x} \)  
3) \( xe^{2x} \)  
4) \( \frac{x^2}{2} e^{2x} \)  

The P.I. of \( (3D^2 + D - 14)y = 13e^{2x} \) is:

1) \( 26xe^{2x} \)  
2) \( 13xe^{2x} \)  
3) \( xe^{2x} \)  
4) \( \frac{x^2}{2} e^{2x} \)
16. \((N, \ast)\) is \(x \ast y = \{ x, y \}\) then \(x, y \in N\) satisfies \((N, \ast)\) is \(\ast\) is

1) associative in \(N\) and \(\ast\) is invertible

2) associative in \(N\) and \(\ast\) is commutative

3) associative in \(N\) and \(\ast\) is commutative and \(\ast\) is invertible

4) commutative.

In \((N, \ast)\), \(x \ast y = \max\{x, y\}\), \(x, y \in N\) then \((N, \ast)\) is

1) only closed

2) only semi-group

3) only monoid

4) a group.

17. 400 students took a Mathematics test. The minimum mark received by the students was 45 and the maximum was 85. The mean was 65. The results show that 120 students scored 85 marks, 85 students scored less than 65 and 45 students scored 65. From these data, determine the number of students who scored 65.

1) 120

2) 20

3) 80

4) 160.

The marks secured by 400 students in a Mathematics test were normally distributed with mean 65. If 120 students got more than 85 marks then the number of students securing marks between 45 and 65 is

1) 120

2) 20

3) 80

4) 160.

18. \(P(X = 0) = k\) and \(P(X = k) = \frac{1}{k}\).

1) \(\log \frac{1}{k}\)

2) \(\log k\)

3) \(e^k\)

4) \(\frac{1}{k}\).

If in a Poisson distribution, \(P(X = 0) = k\), then the variance is

1) \(\log \frac{1}{k}\)

2) \(\log k\)

3) \(e^k\)

4) \(\frac{1}{k}\).
19. The random variable $X$ has mean $\mu_2 = 20$, variance $\mu'_2 = 276$. The mean, variance of random variable $X$ are

1) 16 2) 5 3) 2 4) 1.

If $\mu_2 = 20$, $\mu'_2 = 276$ for a discrete random variable $X$, then the mean of the random variable $X$ is

1) 16 2) 5 3) 2 4) 1.

20. $X$ is a random variable with mean $\mu$, variance $\sigma^2$. $P(a < X < b) =$

1) $P(a \leq X \leq b)$ 2) $P(a < X \leq b)$
3) $P(a \leq X < b)$ 4) $\sigma^2$.

If $X$ is a continuous random variable then $P(a < X < b) =$

1) $P(a \leq X \leq b)$ 2) $P(a < X \leq b)$
3) $P(a \leq X < b)$ 4) all of these.

21. $A$ is a $3 \times 3$ matrix of order 3, which is $det(kA) = k^n$

1) $k^3 det(A)$ 2) $k^2 det(A)$
3) $k det(A)$ 4) $det(A)$.

If $A$ is a matrix of order 3, then $det(kA)$ is

1) $k^3 det(A)$ 2) $k^2 det(A)$
3) $k det(A)$ 4) $det(A)$.

22. $-2x + y + z = l; x - 2y + z = m; x + y - 2z = n$ are such that $l + m + n = 0$.

1) unique solution 2) trivial solution
3) infinitely many solutions 4) no solution.
23. \[
\begin{bmatrix}
\lambda & -1 & 0 \\
0 & \lambda & -1 \\
-1 & 0 & \lambda
\end{bmatrix}
\] 

Let $\lambda$ be an eigenvalue with multiplicity 2. Determine $\lambda$. 

1) $1$  
2) $2$  
3) $3$  
4) $\text{any real number.}$

If the rank of the matrix \[
\begin{bmatrix}
\lambda & -1 & 0 \\
0 & \lambda & -1 \\
-1 & 0 & \lambda
\end{bmatrix}
\] is 2, then $\lambda$ is 

1) $1$  
2) $2$  
3) $3$  
4) $\text{any real number.}$

24. Let $\mathbf{A}$ be an $n \times n$ matrix. If $\mathbf{P}$ is an $n \times n$ matrix such that $\mathbf{P} = \mathbf{P} = 1$, then $\mathbf{P}$ is a 

1) $\text{zero vector}$  
2) $\text{identity matrix}$  
3) $\text{any real number}$  
4) $\text{inconsistent}$

In the system of 3 linear equations with three unknowns, if $\mathbf{P} = \mathbf{P} = 1$, then the system

1) $\text{has unique solution}$  
2) $\text{reduces to 2 equations and has infinitely many solutions}$  
3) $\text{reduces to a single equation and has infinitely many solutions}$  
4) $\text{is inconsistent.}$

25. \[
\vec{a} + \vec{b} + \vec{c} = \vec{0}, \quad |\vec{a}| = 3; \quad |\vec{b}| = 4 \quad \text{and} \quad |\vec{c}| = 5.
\] 

Find $\theta_{\vec{a},\vec{b}}$. 

1) $\frac{\pi}{6}$  
2) $\frac{2\pi}{3}$  
3) $\frac{5\pi}{3}$  
4) $\frac{\pi}{2}$

If $\vec{a} + \vec{b} + \vec{c} = \vec{0}, \quad |\vec{a}| = 3; \quad |\vec{b}| = 4$ and $|\vec{c}| = 5$, then the angle between $\vec{a}$ and $\vec{b}$ is

1) $\frac{\pi}{6}$  
2) $\frac{2\pi}{3}$  
3) $\frac{5\pi}{3}$  
4) $\frac{\pi}{2}$
26. \[ \left( \frac{-1 + i \sqrt{3}}{2} \right)^{100} + \left( \frac{-1 - i \sqrt{3}}{2} \right)^{100} \]

\[ \text{The value of } \left( \frac{-1 + i \sqrt{3}}{2} \right)^{100} + \left( \frac{-1 - i \sqrt{3}}{2} \right)^{100} \text{ is} \]

1) 2  \quad 2) 0  \quad 3) -1  \quad 4) 1.

27. \( ax^2 + bx + 1 = 0 \) is a root of the equation \( ax^2 + bx + 1 = 0 \) where \( a \) and \( b \) are real, then \( (a, b) \) is

1) \((1, 1)\)  \quad 2) \((1, -1)\)  \quad 3) \((0, 1)\)  \quad 4) \((1, 0)\).

28. \( P \) is the point of intersection of \( z \) and \( \text{arg} \( z \) \). \( |2z - 1| = 2|z| \) then the locus of \( P \) is

1) \( x = \frac{1}{4} \) \quad 2) \( y = \frac{1}{4} \) \quad 3) \( z = \frac{1}{2} \) \quad 4) \( x^2 + y^2 - 4x - 1 = 0 \).

If \( P \) represents the variable complex number \( z \) and if \( |2z - 1| = 2|z| \) then the locus of \( P \) is

1) the straight line \( x = \frac{1}{4} \) \quad 2) the straight line \( y = \frac{1}{4} \)

3) the straight line \( z = \frac{1}{2} \) \quad 4) the circle \( x^2 + y^2 - 4x - 1 = 0 \).
29. \( P(x) = 0 \) is a polynomial that has only real roots if and only if it can be expressed as a product of linear factors. Which of the following conditions must be true?

1) It is a polynomial with real coefficients.
2) It is a quadratic polynomial.
3) It is a product of distinct linear factors.
4) It is a product of linear factors with real coefficients.

Polynomial equation \( P(x) = 0 \) admits conjugate pairs of imaginary roots only if the coefficients are

1) Imaginary numbers
2) Complex numbers
3) Real numbers
4) Either real or complex numbers.

30. \( 4x + 2y = c \) where \( c \) is a constant \( y^2 = 16x \) where the parabola intersects \( x \)-axis at \( 0 \). The value of \( c \) is

1) \(-1\)
2) \(-2\)
3) \(4\)
4) \(-4\).

The line \( 4x + 2y = c \) is a tangent to the parabola \( y^2 = 16x \), then \( c \) is

1) \(-1\)
2) \(-2\)
3) \(4\)
4) \(-4\).

31. \( y = 3e^x \) and \( y = \frac{a}{3} e^{-x} \) where the curves intersect at \( (0,0) \). The value of \( a \) is

1) \(-1\)
2) \(1\)
3) \(\frac{1}{3}\)
4) \(3\).

The value of \( a \) so that curves \( y = 3e^x \) and \( y = \frac{a}{3} e^{-x} \) intersect orthogonally is

1) \(-1\)
2) \(1\)
3) \(\frac{1}{3}\)
4) \(3\).
32. \[
\frac{x+1}{x+3} \quad \text{in the limit as } x \to 0 \text{ it is not possible to evaluate the limit directly.}
\]

\[
f(x) = x + 1 \quad \text{and} \quad g(x) = x + 3
\]

1) not continuous
2) not differentiable
3) not in the indeterminate form as \( x \to 0 \)
4) in the indeterminate form as \( x \to 0 \).

L'Hôpital's rule cannot be applied to \( \frac{x+1}{x+3} \) as \( x \to 0 \) because \( f(x) = x + 1 \) and \( g(x) = x + 3 \) are

1) not continuous
2) not differentiable
3) not in the indeterminate form as \( x \to 0 \)
4) in the indeterminate form as \( x \to 0 \).

33. \[
u = \frac{1}{\sqrt{x^2 + y^2}} \quad \text{where} \quad x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}
\]

1) \( \frac{1}{2} \)
2) \( u \)
3) \( \frac{3}{2} \)
4) \( -u \).

If \( u = \frac{1}{\sqrt{x^2 + y^2}} \), then \( x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} \) is

1) \( \frac{1}{2} \)
2) \( u \)
3) \( \frac{3}{2} \)
4) \( -u \).

34. \[
a^2 y^2 = x^2 (a^2 - x^2) \quad \text{in the limit as}
\]

1) \( x = 0 \) \( \text{and} \quad x = a \) \( \text{is}
\]
2) \( x = 0 \) \( \text{and} \quad x = a \) \( \text{is}
\]
3) \( x = -a \) \( \text{and} \quad x = a \) \( \text{is}
\]
4) \( \text{in the limit as}
\]

The curve \( a^2 y^2 = x^2 (a^2 - x^2) \) has

1) only one loop between \( x = 0 \) and \( x = a \)
2) two loops between \( x = 0 \) and \( x = a \)
3) two loops between \( x = -a \) and \( x = a \)
4) no loop.
35. \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) is the equation of an ellipse. The volumes of the solid obtained by revolving the area of the ellipse about major and minor axes are in the ratio

1) \( b^2 : a^2 \)  
2) \( a^2 : b^2 \)  
3) \( a : b \)  
4) \( b : a \).

The volumes of the solid obtained by revolving the area of the ellipse about major and minor axes are in the ratio

1) \( b^2 : a^2 \)  
2) \( a^2 : b^2 \)  
3) \( a : b \)  
4) \( b : a \).

36. \( y = ke^{\lambda x} \) is the solution of the differential equation \( \frac{dy}{dx} = ky \) (where \( k \) is the arbitrary constant). If \( y = ke^{\lambda x} \), then its corresponding differential equation is

1) \( \frac{dy}{dx} = \lambda y \)  
2) \( \frac{dy}{dx} = ky \)  
3) \( \frac{dy}{dx} + ky = 0 \)  
4) \( \frac{dy}{dx} = e^{\lambda x} \).

37. \( \frac{d^2y}{dx^2} = \left[ 4 + \left( \frac{dy}{dx} \right)^2 \right]^\frac{3}{4} \) is the solution of the differential equation \( \frac{d^2y}{dx^2} = \left[ 4 + \left( \frac{dy}{dx} \right)^2 \right]^\frac{3}{4} \) are

1) \( 2, 1 \)  
2) \( 1, 2 \)  
3) \( 2, 4 \)  
4) \( 4, 2 \).

The order and degree of the differential equation \( \frac{d^2y}{dx^2} = \left[ 4 + \left( \frac{dy}{dx} \right)^2 \right]^\frac{3}{4} \) are

1) \( 2, 1 \)  
2) \( 1, 2 \)  
3) \( 2, 4 \)  
4) \( 4, 2 \).
38. $p \iff q$ என்பது என்பது என்பது

1) $p \rightarrow q$

2) $q \rightarrow p$

3) $(p \rightarrow q) \lor (q \rightarrow p)$

4) $(p \rightarrow q) \land (q \rightarrow p)$.

$p \iff q$ is equivalent to

1) $p \rightarrow q$

2) $q \rightarrow p$

3) $(p \rightarrow q) \lor (q \rightarrow p)$

4) $(p \rightarrow q) \land (q \rightarrow p)$.

39. வெளியெடுக்கப்பட்டு முடியைக் கேள்வி முதலை?

1) $p \lor q$

2) $p \land q$

3) $p \lor (\neg p)$

4) $p \land (\neg p)$.

Which of the following is a contradiction?

1) $p \lor q$

2) $p \land q$

3) $p \lor (\neg p)$

4) $p \land (\neg p)$.

40. திறையாரமுணராய் முடி என்று?

1) ஒரு குறிப்பிட்டும் ஒரு குறிப்பிட்டும் குறிப்பிட்டும் குறிப்பிட்டும் குறிப்பிட்டும்

2) குறிப்பிட்டும் குறிப்பிட்டும் குறிப்பிட்டும் குறிப்பிட்டும் குறிப்பிட்டும் குறிப்பிட்டும்

3) பண்பாடுப் பண்பாடுப் பண்பாடுப் பண்பாடுப் பண்பாடுப் பண்பாடு

4) பண்பாடு $a, b \in G$ என்றால் $(a \cdot b)^{-1} = a^{-1} \cdot b^{-1}$ [ $G$ இவ்வாறு தோன்றும் ]

Which of the following is correct?

1) An element of a group can have more than one inverse

2) If every element of a group is its own inverse, then the group is Abelian

3) The set of all $2 \times 2$ real matrices forms a group under matrix multiplication

4) $(a \cdot b)^{-1} = a^{-1} \cdot b^{-1}$ for all $a, b \in G$, a group.
Prove that \((AB)^{-1} = B^{-1} A^{-1}\) where \(A\) and \(B\) are two non-singular matrices.

\[
\begin{bmatrix}
0 & 1 & 2 & 1 \\
2 & -3 & 0 & -1 \\
1 & 1 & -1 & 0
\end{bmatrix}
\]

Find the rank of the matrix \[
\begin{bmatrix}
0 & 1 & 2 & 1 \\
2 & -3 & 0 & -1 \\
1 & 1 & -1 & 0
\end{bmatrix}
\]

43. i) \(\vec{a}, \vec{b}\) என்பன | ையை எந்த எய்களுக்கு முதலில் \(|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2\) என்றோ என்பொருளாக.

ii) \(2x - y + z = 4 \) மற்றும் \(x + y + 2z = 4\) என்பன எங்குள் கேட்குதோன்றக்கு மேலும் கேட்குதோன்றக்கு

iii) If \(\vec{a}, \vec{b}\) are any 2 vectors, then prove that \(|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2.\)

iv) Find the angle between the planes \(2x - y + z = 4\) and \(x + y + 2z = 4.\)
44. \(2 \mathbf{i} + 6 \mathbf{j} - 7 \mathbf{k}\) and \(2 \mathbf{i} - 4 \mathbf{j} + 3 \mathbf{k}\) are the position vectors of points \(A\) and \(B\) respectively. Find the equation of the sphere on the join of the points \(A\) and \(B\) having position vectors \(2 \mathbf{i} + 6 \mathbf{j} - 7 \mathbf{k}\) and \(2 \mathbf{i} - 4 \mathbf{j} + 3 \mathbf{k}\) respectively as a diameter.

45. Let the line \(z\) cut \(x, y, z\) axes at \(A, B, C\) respectively. If \(P\) is a variable point on the line, then

\[
|3z - 5| = 3|z + 1|
\]

Find the locus of \(P\) if

\[
|3z - 5| = 3|z + 1|
\]

46. \(\cos \alpha + \cos \beta + \cos \gamma = 0 = \sin \alpha + \sin \beta + \sin \gamma\),

\[\cos 2\alpha + \cos 2\beta + \cos 2\gamma = 0\]

\[\sin 2\alpha + \sin 2\beta + \sin 2\gamma = 0\]

If \(\cos \alpha + \cos \beta + \cos \gamma = 0 = \sin \alpha + \sin \beta + \sin \gamma\), prove that

\[\cos 2\alpha + \cos 2\beta + \cos 2\gamma = 0\]

\[\sin 2\alpha + \sin 2\beta + \sin 2\gamma = 0\]
A standard R.H has its vertices at (5, 7) and (-3, -1). Find its equation and asymptotes.

At 2:00 p.m., a car's speedometer reads 30 miles/hr, and at 2:10 p.m., it reads 50 miles/hr. Show that sometime between 2:00 p.m. and 2:10 p.m., the acceleration is exactly 120 miles/hr².

Find the absolute maximum and minimum values of the function

\[ f(x) = x^3 - 3x^2 + 1, \quad \frac{-1}{2} \leq x \leq 4 \]
50. \( U = (x - y)(y - z)(z - x) \) then \( \frac{U_x + U_y + U_z}{= 0} \) holds if \( x \neq y \neq z \).

If \( U = (x - y)(y - z)(z - x) \), then show that \( U_x + U_y + U_z = 0 \).

51. Evaluate:

\[
\begin{align*}
\text{i)} \quad & \int_{0}^{1} \frac{1}{\sqrt{4 - x^2}} \, dx \\
\text{ii)} \quad & \int_{0}^{\frac{\pi}{2}} \sin^6 x \, dx.
\end{align*}
\]

52. \( (x^2 + y^2) \, dy = xy \, dx \).

Solve: \( (x^2 + y^2) \, dy = xy \, dx \).

53. Prove that the set of all 4th roots of unity forms an Abelian group under multiplication.
54. A game is played with a single fair die. A player wins Rs. 20 if 2 turns up, Rs. 40 if 4 turns up and loses Rs. 30 if 6 turns up, while he neither wins nor loses if any other face turns up. Find the expected sum of money he can win.

55. a) \[ p \leftrightarrow q = ((\neg p) \lor q) \land ((\neg q) \lor p) \] 

b) Suppose that the amount of cosmic radiation to which a person is exposed when flying by jet across the United States is a random variable having a normal distribution with a mean of 4.35 m rem and a standard deviation of 0.59 m rem. What is the probability that a person will be exposed to more than 5.20 m rem of cosmic radiation of such a flight?
56. A small seminar hall can hold 100 chairs. Three different colours (red, blue and green) of chairs are available. The cost of a red chair is Rs. 240, cost of a blue chair is Rs. 260 and the cost of a green chair is Rs. 300. The total cost of chairs is Rs. 25,000. Find at least 3 different solutions of the number of chairs in each colour to be purchased [use determinant method].

57. \( \cos (A + B) = \cos A \cos B - \sin A \sin B \) and \( \text{prove that } \cos (A + B) = \cos A \cos B - \sin A \sin B \) by vector method.
58. \[
\frac{x - 1}{1} = \frac{y + 1}{-1} = \frac{z}{3} = \frac{x - 2}{1} = \frac{y - 1}{2} = \frac{-z - 1}{1}
\]

Show that the lines \[
\frac{x - 1}{1} = \frac{y + 1}{-1} = \frac{z}{3} \text{ and } \frac{x - 2}{1} = \frac{y - 1}{2} = \frac{-z - 1}{1}
\]
intersect and find their point of intersection.

59. \[x^9 + x^5 - x^4 - 1 = 0\]

Solve the equation: \[x^9 + x^5 - x^4 - 1 = 0\].

60. \[x^2 - 4x + 4y = 0\]

Find the axis, vertex, focus, equations of directrix and latus rectum and length of the latus rectum of the parabola \[x^2 - 4x + 4y = 0\] and hence draw the diagram.

51. \[\begin{align*}
\text{Given:} & \quad \text{Equations:} & \quad (2, 4) & \quad \text{and} & \quad (2, 0)\\
\text{Find:} & \quad \text{Solve:} & \quad x + 2y - 12 = 0 & \quad \text{and} & \quad x - 2y + 8 = 0
\end{align*}\]

Find the equation of the hyperbola if its asymptotes are parallel to \[x + 2y - 12 = 0\]
and \[x - 2y + 8 = 0\]. Further \((2, 4)\) is the centre of the hyperbola and it passes through \((2, 0)\).
62. Two sides of a triangle have length 12 m and 15 m. The angle between them is increasing at a rate of 2°/min. How fast is the length of the third side increasing when the angle between the sides of fixed length is 60°?

63. Find the dimensions of the rectangle of the largest area that can be inscribed in a circle of radius $r$.

64. Trace the curve $y^2 = 2x^3$.

65. Find the area bounded by $x$-axis and an arch of the cycloid $x = a(2t - \sin 2t)$; $y = a(1 - \cos 2t)$.

66. Find the length of the curve \[ \left( \frac{x}{a} \right)^2 + \left( \frac{y}{a} \right)^2 = 1 \]
67. \( \frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + 2y = 2e^{3x} \) when \( x = \log 2 \).

Solve the differential equation \( \frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + 2y = 2e^{3x} \) when \( x = \log 2, y = 0 \) and when \( x = 0, y = 0 \).

68. \((\mathbb{Z}_n, +_n)\) காண்க குழு காண்க.

Show that \((\mathbb{Z}_n, +_n)\) forms a group.

69. கீழே வரைபடமான மாறி \( X \) காட்டு விளக்கத்தை அறிவிக்கும் காரண

\[ f(x) = \begin{cases} kx^{\alpha-1}e^{-\beta x^\alpha}, & x, \alpha, \beta > 0 \\ 0, & \text{otherwise} \end{cases} \]

ii) \( P( X > 10 ) \) காண்க.

The probability density function of a random variable \( X \) is

\[ f(x) = \begin{cases} kx^{\alpha-1}e^{-\beta x^\alpha}, & x, \alpha, \beta > 0 \\ 0, & \text{elsewhere} \end{cases} \]

Find

i) \( k \)

ii) \( P( X > 10 ) \).
70. a) The arch of a bridge is in the shape of a semi-ellipse having a horizontal span of 40 ft and 16 ft high at the centre. How high is the arch, 9 ft from the right or left of the centre?

OR

b) For a postmortem report, a doctor requires to know approximately the time of death of the deceased. He records the first temperature at 10:00 am to be 93.4°F. After 2 hours, he finds the temperature to be 91.4°F. If the room temperature (which is constant) is 72°F, estimate the time of death. (Assume normal temperature of a human body to be 98.6°F.)